This month’s column, the second in a series of three on compressible flow, derives the key relationships for understanding steady, isentropic flow in a converging-diverging nozzle. The starting point of these derivations are the three isentropic jump conditions derived in the last post:

\[ \begin{array}{ll} {\textrm{continuity:}}& {\rho}\_1 V\_1 A\_1 = {\rho}\_2 V\_2 A\_2 \equiv {\dot m}\\{\textrm{momentum:}}& (\rho\_1 V\_1^2 + P\_1) A\_1 = (\rho\_2 V\_2^2 + P\_2) A\_2 \\{\textrm{energy:}}& h\_1 + \frac{{V\_1}^2}{2} = h\_2 + \frac{{V\_2}^2}{2} \end{array} \; , \]

along with the first law of thermodynamics (cast in intensive variables)

\[ de = T ds + \frac{P}{\rho^2} d \rho \; ,\]

the equation of state for an ideal gas

\[ P \rho = R T \; , \]

and the fact that the specific internal energy and specific enthalpy are simply functions of temperature given by $$e = c\_{{\mathcal V}} T$$ and $$h = c\_P T$$, respectively.

There are three ingredients in understanding the [steady, isentropic flow through a converging-diverging nozzle](https://en.wikipedia.org/wiki/Isentropic_nozzle_flow) (also known as a [de Laval nozzle](https://en.wikipedia.org/wiki/De_Laval_nozzle)): 1) calculating the Mach number, 2) relating the thermodynamics variables of temperature ($$T$$), the pressure ($$P$$) and the density ($$\rho$$) to the Mach number, and 3) relating the cross-sectional area of the nozzle ($$A$$) to the Mach number.

This post is a synthesis of the found in Chapter 9 of Merle Potters *Fluid Mechanics Demystified* and the 5-part series of YouTube lectures on compressible fluid flow from the University of Florida ([part 1](https://www.youtube.com/watch?v=CnAEwDQxeKo), [part 2](https://www.youtube.com/watch?v=ZzyU7BHCllY), [part 3](https://www.youtube.com/watch?v=o-UGasX1atc), [part 4](https://www.youtube.com/watch?v=MdYYgyHNDFY), and [part 5](https://www.youtube.com/watch?v=L-6gvVuiAQ4))

Calculating the Mach Number